IMPROVING RELIABILITY OF VIBRATION SPECTRUM IDENTIFICATION WITHOUT INCREASING SAMPLING RATE USING SPECTRAL INTERPOLATION ALGORITHM

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Condition diagnostics and monitoring of machinery by vibration-acoustic diagnostics methods is widely used nowadays in production industries, in order to quit the expensive preventive maintenance and implement an effective modern system of condition-based maintenance. The papers [1,2] show that the effectiveness of a condition monitoring system implementation is mainly determined by the value of the error of the result issued by the expert system. One of the components of the diagnostic errors is low resolution power of spectral analyzer which is currently the main instrument for vibration-acoustic signal analysis.

Resolution of spectral analyzer for a discreet digital signal can be found by the following formula:

$$\Delta F = \frac{F_S}{N_S}, \text{ where} \tag{1}$$

Fs – sampling frequency;
Ns – number of samples (sample rate).

According to the formula 1 a standard approach towards increasing resolution comprises reducing sampling frequency and increasing number of samples.

However, reducing sampling frequency, as stated by the sampling theorem, leads to decrease of the higher threshold of the signal's measured frequency range. Also, an increase of sampling rates lowers the system's response speed, which is highly important for stationary automatic monitoring systems, since it may lead to an increase of dynamic component of a diagnostic error.

Thus, the development of methods for increase of resolution power of spectral analyzer without changing the sampling rate parameters is an urgent task.

One of such methods is using an interpolation algorithm in order to obtain adjusted values of the vibration signal's spectral component frequencies [3]. The method is based on adjusting the spectral component frequencies by applying the window interpolation function on amplitudes of the closest frequency components. The principle of the method's work is shown on fig.1. In the center there is a part of the harmonic spectrum with its frequency matching with the central bandwidth of the spectrum (100 Hz, bandwidth – 1Hz). On the right and left sides there are the spectrum's parts when the harmonic frequency is shifted on a half of the spectrum bandwidth up or down (F=99.5 and 100.5 Hz). It can be seen, that in that case the amplitudes of the closest frequency components are changing, and the papers [3,4] present analytic formulae for adjusting the spectral component frequencies of the closest components. Thus, when using Hann Window, the adjusted harmonic *Fp* will be calculated by the formula:

$$F_P = F_0 \ \pm \frac{2-K_P}{1+K_P} \cdot \Delta F, \text{ where} \tag{2}$$

F_0 – central frequency of the spectrum bandwidth;
ΔF – spectrum bandwidth;
Kp – relation between an amplitude of the central spectrum bandwidth and amplitude of the maximum closest band.
The sign (plus or minus) in the formula 2 depends on the place of the maximum closest band in relation to the central one.

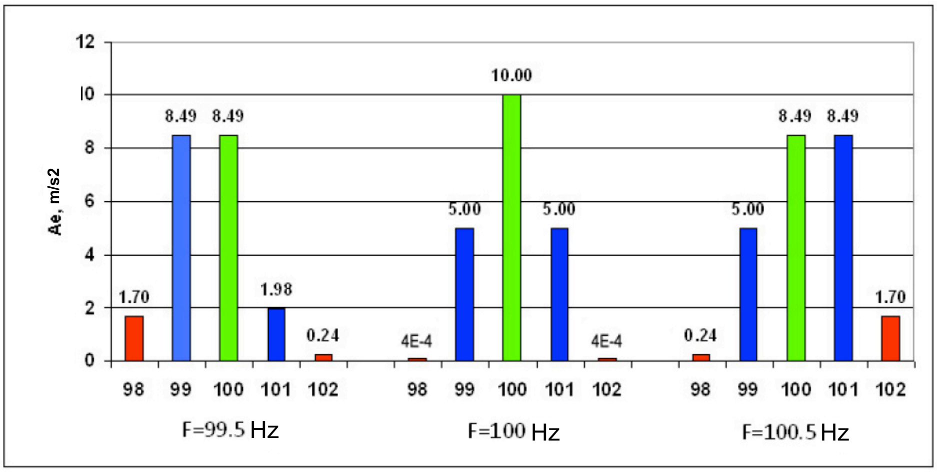

Figure 1 – Operation principle of the interpolation method

The main disadvantage of the method is a dependency between the accuracy of assessments and the signal's noise component. Figure 2 shows a dependency between harmonic frequency's interpolation evaluation error and signal-noise relation (SNR) for the sinusoidal waveform [4]. The error in this case was calculated in % depending on the bandwidth of the spectrum analyzer.

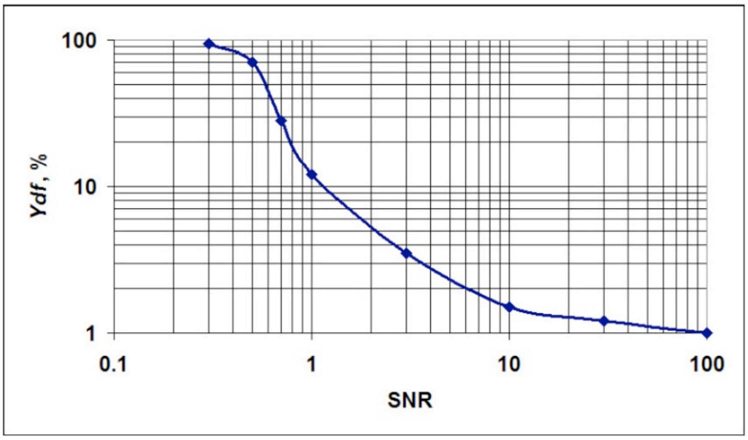

Figure 2 - Dependency between harmonic frequency's interpolation evaluation error and signal-noise relation for the sinusoidal waveform

It may be noted that, when SNR is low, the error goes to 100% (to the bandwidth of the spectrum analyzer) and when SNR grows the error lowers with a threshold effect when SNR =1. Theoretically, when there is no noise, the method's error will be determined by spectrum frequency component amplitudes measuring error.

The given algorithm is successfully applied in monitoring and diagnostic systems, such as the ones enlisted in [1.2] using a special sin-cursor option in a spectral analysis. However, the dependency between the method's error and the noise level leads to necessity to evaluate the real noise rate and consider it in finalizing the assessment. Currently, it is done manually, and this paper is devoted to developing the method of automatic assessment of interpolation method error due to SNR.

When the noise rate is known, to solve this task one needs an inverse dependence (as shows on fig.2) and calculation of the obtained error by that dependence. However, the assessment of a real spectrum's noise component rate is a nontrivial task. Also, to make an error's assessment according to the given dependency (fig.2) one needs to evaluate the noise level around the analyzed spectrum harmonics.

The SNR around the harmonics can be evaluated by the following formula:

$$K_{SNR} = \frac{A_0}{\sqrt{{A_{-2}}^2 + {A_{+2}}^2}}, \textit{where} \tag{3}$$

A_0 – an amplitude of the central (maximum) harmonic component (Fig. 1);
A_{-2} – an amplitude of the second left harmonic component from the central (maximum) one;
A_{+2} – an amplitude of the second right component from the central (maximum) one;
The point of the formula is that SNR is evaluated by the common level of the frequency components following the closest ones, and those, whose amplitude is not sufficiently affected by the window function. However, more detailed consideration of the formula 3 reveals that the formula is invalid, since when the harmonic frequency is shifted from the spectrum bandwidth center, the level of the closest side harmonics is changing depending on to

which side the frequency is shifted in relation to center. Fig 2 shows that when the harmonic signal's frequency is shifted from the center of the bandwidth (100 Hz) down (99.5 Hz) or up (100.5 Hz) and there is no noise, the corresponded side component intended for SNR evaluation (in red) increases.

Thus, the common level of the side harmonics will be increasing anyway if the real frequency is shifted from the FFT bandwidth center, and on the sides of the bandwidth will be taken the values comparable to the main harmonics only, which makes it impossible – to evaluate the noise rate.

To eliminate the impact of this effect it is suggested to use the rates of an only one *minimal* harmonic instead of the combined rates of the both harmonics. In that case, the frequency's shift from the center causes its level to reduce and, consequently, reduce impact on noise assessment.

Therefore, to make an assessment of SNR by the spectral level around the focused harmonic component, the following formula is used:

$$K_{SNR} = \frac{A_0}{\min(A_{-2}, A_{+2})}, where \tag{3}$$

min – function for calculating minimal value from A_{-2} $A_{+2.}$

SNR value calculated by the formula 3 will differ from a real SNR because of the impact – although the lower one - by the window function. That is why to obtain frequency assessment error dependency from K_{SNR} evaluation a numerical experiment was conducted using Monte Carlo method. In Agilent Vee a combination of sine-wave signal (RMSV Us, frequency Fs and a sporadic signal (RMSV Uh) entered the frequency assessment calculating unit Fi using interpolation method by formula 2 and at the same time calculating K_{SNR} by formula 3.

Further, a real, adjusted to the spectral bandwidth, frequency assessment error was calculated by the formula:

$$dF = \frac{|Fs-Fi|}{Fw}, where \tag{4}$$

Fw – is the spectral bandwidth, Hz.

Thus, the experiment resulted in paired-values array of the real frequency assessment error and K_{SNR} calculated by formula 3 for different real SNR values. The real SNR value varied in the 0 - 50 range in increments of 1 and in every value 10000 measurements of error dF (formula 4) and K_{SNR} (formula 3) were made. It should be noted, that in each point of real SNR, the sine-wave signal frequency were changing sporadically within the spectral bandwidth.

The experiment's results are given on Fig. 3.

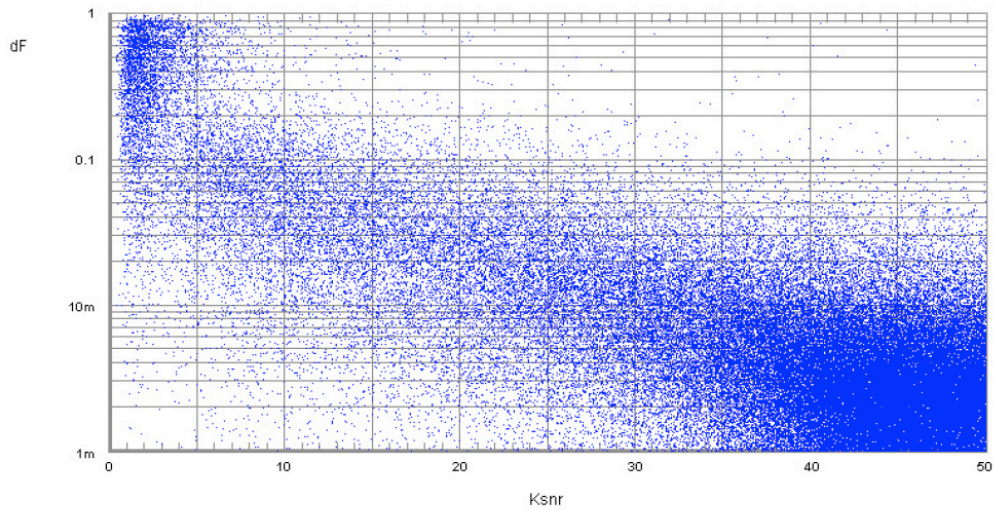

Fig 3 – Results of experiment based on Monte Carlo method aimed at obtaining frequency assessment error dependency from K_{SNR} evaluation for a sine-wave signal

Each point on the graph is a pair of dF and K_{SNR} values. It should be noted, that when K_{SNR} has a particular value, the dF error values distributed sporadically, and mode of the distribution is increases together with K_{SNR} value. Since we are interested in maximum potential error value of frequency assessment, we calculated dF error per each set K_{SNR} value at 0.99 probability level.

Fig. 4 shows frequency assessment error dF dependency from K_{SNR} noise coefficient for 0.99 probability level (blue line) and its approximation with 4 splines (purple line).

Spline approximation consists of 4 areas:

- limiting area with Ksnr < 8 dF = 1;

- linear area with 8 < Ksnr > 15;
- logarithmic linear area 8 < Ksnr > 15;
- limiting area Ksnr > 46 dF = 0.03.

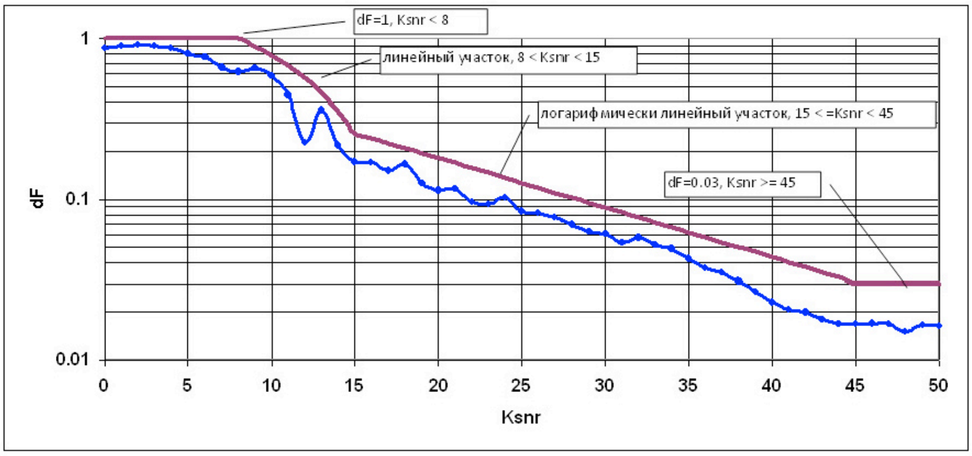

Figure 4 - Frequency assessment error dF dependency from K_{SNR} noise coefficient for 0.99 probability level

Formula (5) describes resulting analytic expression for calculation of frequency assessment error adjusted to spectral bandwidth using the SNR value.

$$dF = \begin{cases} 1, & \text{if} \quad K_{snr} < 9 \\ -0.0108 \cdot K_{snr} + 1.864, & \text{if} \quad 9 \le K_{snr} < 15 \\ 10^{(-0.031 \cdot K_{snr} - 0.122)}, & \text{if} \quad 15 \le K_{snr} < 45 \\ 0.03, & \text{if} \quad K_{snr} \ge 45 \end{cases} \tag{5}$$

Using the given error value, the actual error of frequency assessment δF can be calculated by the following formula:

$$\delta F = dF \cdot Fw, where \tag{6}$$

Fw – is the spectral bandwidth, Hz.

The developed algorithm was tested in Agilent Vee as well, by means of numerical experiment identical to the one used for obtaining formula 5. A combination of sine-wave and sporadic signals entered the frequency assessment calculating unit dF by formulas 5, 6 and actual error calculation unit by formula 4. Further, using obtained array, we calculated a percentage rate of times when error assessment value of an actual error was exceeded, for each SNR value.

The experiment result is given on fig.5, which shows that set probability level is slightly exceeded the evaluating interval (1.1% instead of 1%) when Ksnr is within 20 – 27 range. In all other areas of Ksnr range the error level is less than 1%. That confirms the reliability of the developed method for frequency error assessment.

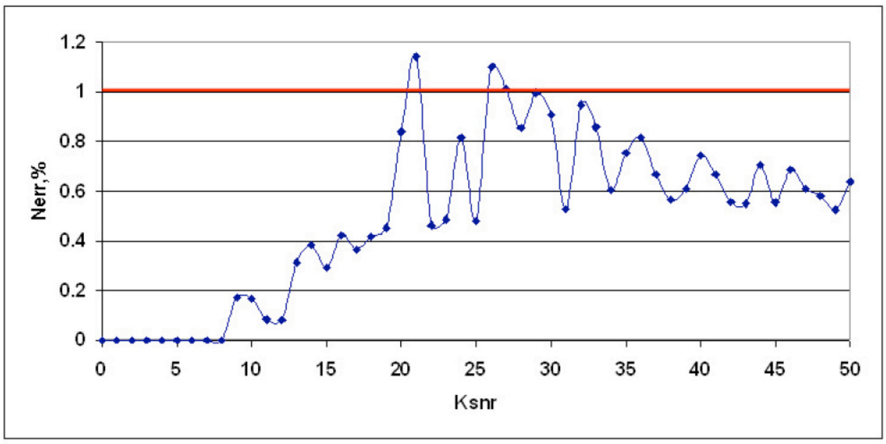

Figure 5 – Percentage dependency of a number of errors (Nerr) for frequency error assessment from different KSNR values

Conclusions

1. The described method for error assessment of interpolation value of frequency allows to evaluate the interpolation method accuracy when SNR value is in 0-46 range. When the values are high, this method gives overestimated assessment (set value is 0.03 from the bandwidth) due to side lobes of the window function in case of low noise level.

2. The algorithm testing revealed that maximum probability of an actual frequency exceeding the estimating interval is 1.1%.

3 the method can be applied in systems for vibration acoustic diagnostics and allows to carry out more accurate detection of frequency components of defects, while analyzing vibration spectrum, and, consequently, to reduce diagnostic errors.

References

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